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On the Calculation of Survivorship Annuities by the Columnar Method. By JAMES MEIKLE, *Esq., of the Scottish Provident Institution, Edinburgh.*

[Read before the Institute, 26th January, 1863, and printed by order of the Council.]

THE assurance of a capital sum payable at the death of a husband is of much more recent adoption than the provision of an annuity during the life of his widow. Some of the earliest Assurance Companies were instituted with the main design of providing annuities to widows and sisters—now, however, forming a very small portion of their business—and very many funds and schemes, instituted at a very early period, are now in existence, throughout the various sections, professions and trades, of the community, whose only end is to secure annuities commencing on the death of the person upon whose income the annuitant may be supposed to be dependant. The assurance of a capital sum payable at death seems specially fitted for the repayment of an advance, or as a security for its repayment, and seems inapplicable as an assurance for the benefit of the family where the wife may die before the husband without family, or where the family may grow up and be as well off as their father. An annuity provision has, however, many advantages over the assurance of a capital sum, and, indeed, seems to be by far the more natural and the more suitable arrangement for providing a continuance of family support on the death of the husband during the life of his widow or children. A capital sum is endangered by temptations to spend it, while an annuity, payable half-yearly or quarterly, secures a *certainty* of support, frugally perhaps, to the very latest period of the life of the annuitant. The investment of a capital sum—that its return may be applied for the uses of the family—is attended with many disadvantages to the annuitant, such as the fluctuations of the dividend and the uncertainty of the security, over and above the small dividend receivable where the capital is only lent. In regard to the advantages to the Office, the annuity transaction is as safe as the ordinary assurance of a capital sum. At the outset there may be extra risk in a good life being set against a bad one; but this may, in other cases, be more than compensated by the circumstance of an inferior life becoming only an annuitant, in place of being the recipient of a capital sum. There is an apparent disadvantage to the Office in the annuity transaction being carried on during the continuance of *two* lives, whereas the assurance would terminate with the life of one; but it will be remembered that the annuity would be continued during two lives only when it became exigible, for the transaction would terminate with the *joint* existence if the proposed annuitant died first. There are many other external differences between these two systems of family provision, but many of the same arrangements and facilities which have grown out of the one system, by reason of its having been more generally promoted amongst the public, could easily be engrafted upon the other. Many special arrangements might be applied if the annuity system become more general. The unmarried might contribute a smaller premium until marriage—which, in the event of their remaining unmarried, might form the nucleus of a deferred annuity. The greatest merit of the assurance, however, is its apparent convenience and simplicity, whereas the greatest inducement of the annuity, besides its special fitness as a family provision, is its comparative economy. The premium usually required from a person of age 40, for an assurance of £500, is £15, which would secure an annuity of about £50 during the life of his widow, whose present age is 35—being thus equal to 10 per cent. upon the corresponding capital sum, without the risk and the trouble of investment. The annuity might be increased from the profits, in the same way as the sum assured is increased by bonuses; and if an annuity be provided to the family, or to each child, while under 14 or 18 years of age, commencing either on death of the father or on death of both father and mother, the additional payment would not be much. The business of the Life Assurance Companies is almost entirely confined to the assurance of capital sums. Some refuse to grant any annuities, and very few promote extensively the system of *survivorship* annuities. Recently, however, the Mutual Assurance Company in New York—the largest American Office—has developed a complete system of survivorship annuities.

The calculation of the single premium required to provide an annuity to x after z is of a very simple nature, it is the difference between the value of the annuity during the life of the annuitant x and the value of the annuity during the joint existence of the two persons x and $z = (A_x - A_{x.z})$. Such is the calculation in the simplest, and perhaps most frequent, form of the transaction. It assumes the risk to endure until either life fail, when it either becomes a claim for the annuity, or vanishes altogether by the death of the annuitant. When, however, the risk is to endure during a period of years—that is, an annuity is to be payable during the life of x, provided z die in n years—the calculation is somewhat longer, and has hitherto been performed by aid of the following formula—

$${}_{\overline{n|}}\mathrm{A}_x - {}_{\overline{n|}}\mathrm{A}_{x.z} + {}_n a_x v^n \mathrm{A}_{x+n}(1 - {}_n a_z) \text{ (Milne's notation),}$$

${}_n a_x$ being the probability of x living n years,
v^n the present value of £1 due in n years,
A_x the whole-life annuity on x,
${}_{\overline{n|}}\mathrm{A}_x$ the temporary annuity on x for n years;

in which ${}_{\overline{n|}}\mathrm{A}_x - {}_{\overline{n|}}\mathrm{A}_{x.z}$ is the temporary annuity during n years to such of lives aged x who have survived z, and ${}_n a_x v^n \mathrm{A}_{x+n}(1 - {}_n a_z)$ is the deferred annuity commencing in n years to such of the lives aged x who have survived n years, provided z have died within the n years.

No successful attempt has hitherto been made (so far as I am aware) to bring the calculation of survivorship annuities under the columnar system of operation, by which the value of temporary deferred and increasing benefits may be obtained with the greater facilities common to that system. This I now propose to do.

Limiting the foregoing expression to the period of *one year*, it becomes

$$\begin{aligned}
&{}_{\overline{n|}}\mathrm{A}_x - {}_{\overline{n|}}\mathrm{A}_{x.z} + {}_1 a_x v^1 \mathrm{A}_{x+1}(1 - {}_1 a_z) \\
&= \frac{vl_{x+1}}{l_x} - \frac{vl_{x+1}\,l_{z+1}}{l_x.l_z} + \frac{vl_{x+1}\mathrm{A}_{x+1}}{l_x} - \frac{vl_{x+1}\,l_{z+1}}{l_x.l_z}\mathrm{A}_{x+1} \\
&= \frac{vl_{x+1}}{l_x}(1 + \mathrm{A}_{x+1})\left(1 - \frac{l_{z+1}}{l_z}\right) = \frac{\mathrm{A}_x d_z}{l_z} = \frac{l_x \mathrm{A}_x d_z}{l_x l_z}.
\end{aligned}$$

Limited to *two years*, it becomes

$$\begin{aligned}
&{}_{\overline{2|}}\mathrm{A}_x - {}_{\overline{2|}}\mathrm{A}_{x.z} + {}_2 a_x v^2 \mathrm{A}_{x+2}(1 - {}_2 a_z) \\
&= \frac{vl_{x+1}}{l_x} + \frac{v^2 l_{x+2}}{l_x} - \frac{vl_{x+1}\,l_{z+1}}{l_x.l_z} - \frac{v^2 l_{x+2}\,l_{z+2}}{l_x.l_z} + \frac{v^2 l_{x+2}}{l_x}\mathrm{A}_{x+2} - \frac{v^2 l_{x+2}\,l_{z+2}\mathrm{A}_{x+2}}{l_x.l_z} \\
&= \frac{vl_{x+1}}{l_x}\left(1 - \frac{l_{z+1}}{l_z}\right) + \frac{v^2 l_{x+2}}{l_x}(1 + \mathrm{A}_{x+2})\left(1 - \frac{l_{z+2}}{l_z}\right) \\
&= \frac{vl_{x+1}d_z}{l_x.l_z} + \frac{vl_{x+1}\mathrm{A}_{x+1}(d_z + d_{z+1})}{l_x.l_z} \\
&= \frac{vl_{x+1}(1 + \mathrm{A}_{x+1})d_z}{l_x.l_z} + \frac{vl_{x+1}\mathrm{A}_{x+1}\,d_{z+1}}{l_x.l_z} \\
&= \frac{l_x \mathrm{A}_x d_z}{l_x l_z} + \frac{vl_{x+1}\mathrm{A}_{x+1}\,d_{z+1}}{l_x.l_z}
\end{aligned}$$

Limited to *three years,* it becomes

$$ {}_{\overline{3}|}\mathrm{A}_x - {}_{\overline{3}|}\mathrm{A}_{x.z} + {}_3a_x v^3 \mathrm{A}_{x+3}(1 - {}_3a_z) $$

$$ = \frac{vl_{x+1}}{l_x} + \frac{v^2l_{x+2}}{l_x} + \frac{v^3l_{x+3}}{l_x} - \frac{vl_{x+1}\,l_{z+1}}{l_x.l_z} - \frac{v^2l_{x+2}\,l_{z+2}}{l_x.l_z} - \frac{v^3l_{x+3}\,l_{z+3}}{l_x.l_z} + $$

$$ + \frac{v^3l_{x+3}}{l_x}\mathrm{A}_{x+3} - \frac{v^3l_{x+3}\,l_{z+3}}{l_x.l_z}\mathrm{A}_{x+3} $$

$$ = \frac{vl_{x+1}d_z}{l_x.l_z} + \frac{v^2l_{x+2}}{l_xl_z}(d_z + d_{z+1}) + \frac{v^3l_{x+3}}{l_xl_z}(d_z + d_{z+1} + d_{z+2})(1 + \mathrm{A}_{x+3}) $$

$$ = \frac{vl_{x+1}d_z}{l_x.l_z} + \frac{v^2l_{x+2}}{l_x.l_z}(1 + \mathrm{A}_{x+2})(d_z + d_{z+1}) + \frac{v^2l_{x+2}\mathrm{A}_{x+2}d_{z+2}}{l_x.l_z} $$

$$ = \frac{vl_{x+1}(1 + \mathrm{A}_{x+1})d_z}{l_x.l_z} + \frac{vl_{x+1}\mathrm{A}_{x+1}d_{z+1}}{l_x.l_z} + \frac{v^2l_{x+2}\mathrm{A}_{x+2}d_{z+2}}{l_x.l_x} $$

$$ = \frac{l_x\mathrm{A}_xd_z}{l_x.l_z} + \frac{vl_{x+1}\mathrm{A}_{x+1}d_{z+1}}{l_x.l_z} + \frac{v^2l_{x+2}\mathrm{A}_{x+2}d_{z+2}}{l_x.l_z}, $$

from which the law of progression is manifest; therefore the usual formula,

$$ {}_{\overline{n}|}\mathrm{A}_x - {}_{\overline{n}|}\mathrm{A}_{x.z} + {}_na_x v^n \mathrm{A}_{x+n}(1 - {}_na_z), $$

for the present value of an annuity of £1 during life of x provided z die within n years, will be equal to the following expression—

$$ = \frac{l_x\mathrm{A}_xd_z + vl_{x+1}\mathrm{A}_{x+1}d_{z+1} + v^2l_{x+2}\mathrm{A}_{x+2}d_{z+2}}{l_x.l_z} + \text{\&c. to } n \text{ terms.} $$

This expression may, for practical purposes, be most conveniently put into the following form, by multiplying by v^x and converting $v^xl_x\mathrm{A}_x$ into N_x, and representing the numerator by $\mathrm{N}_{\frac{1}{xz}}$.

$$ \frac{\mathrm{N}_{\frac{1}{x.z}}}{\mathrm{D}_xl_z} = \frac{d_z\mathrm{N}_x + d_{z+1}\mathrm{N}_{x+1} + \text{\&c. } \infty}{\mathrm{D}_xl_z} = \mathrm{A}_x - \mathrm{A}_{x.z}. $$

$$ \frac{\mathrm{N}_{\frac{1}{x+n.z+n}}}{\mathrm{D}_{x+n}l_{z+n}} = \mathrm{A}_{x+n} - \mathrm{A}_{x+n.z+n} $$

$$ \frac{\mathrm{N}_{\frac{1}{x.z}} - \mathrm{N}_{\frac{1}{x+n.z+n}}}{\mathrm{D}_x.l_z} = {}_{\overline{n}|}\mathrm{A}_x - {}_{\overline{n}|}\mathrm{A}_{x.z} + {}_na_x v^n \mathrm{A}_{x+n}(1 + {}_na_z) $$

$=$ the present value of an annuity to x after death of z, provided z dies in n years.

Corresponding formulæ for other problems may be reared by following the usual law of the ordinary columnar system.

Examples from the tables appended.

1. Value of annuity to 55 after 65 :—

$\dfrac{N_{\overset{1}{\overline{55.65}}}}{D_{55}.l_{65}} = (6,400,563)\ \lambda\ 6{\cdot}8062182$

$D_{55}\ \kappa\ \bar{3}{\cdot}3269193$

$l_{65}\ \kappa\ \bar{4}{\cdot}5202808$

$\lambda\ {\cdot}6594183 = 4{\cdot}5021$

which agrees with the usual method,

$A_{55} = 11{\cdot}29961$

$A_{55.65}$ (Jones) $6{\cdot}79812$

$= 4{\cdot}50149$

In the following examples the advantages of the columnar method will be observed.

2. Value of annuity to 50 after 60, provided 60 die in 7 years :—

$N_{\overset{1}{\overline{50.60}}} = 10,678,027$

$N_{\overset{1}{\overline{57.67}}} = 5,140,533$

Log. of $5,537,494 = \lambda\ 6{\cdot}7433137$

$D_{50}\ \kappa\ \bar{3}{\cdot}2085105$

$l_{60}\ \kappa\ \bar{4}{\cdot}4385408$

$\lambda\ {\cdot}3903650 = 2{\cdot}457$

while by the usual method (very much abridged),

${}_{\overline{7}|}A_{50} = 5{\cdot}6631$

${}_{\overline{7}|}A_{50.60} = N_{51.61} = 12,382,627$

$N_{58.68} = 4,873,982$

$\lambda\ 6{\cdot}8755615 = 7,508,645$

$D_{50.60}\ \lambda\ 6{\cdot}1826143 = 1,522,703$ (Chisholm's tables).

${\cdot}6929472 = 4{\cdot}9311$

${}_{\overline{7}|}A_{50} - {}_{\overline{7}|}A_{50.60} = {\cdot}7320$

$D_{57} = \lambda\ 2{\cdot}6228287$

$D_{50} = \kappa\ \bar{3}{\cdot}2085105$

$A_{57} = \lambda\ 1{\cdot}0263530$

$l_{60} = 3643$

$l_{67} = 2771$

$872\quad \lambda\ 2{\cdot}9405165$

$l_{60} = \kappa\ \bar{4}{\cdot}4385408$

${}_{7}av^{7}.A_{x+7}(1 - {}_{7}a_{x}) = \lambda\ {\cdot}2367495 = 1{\cdot}7248$

$= 2{\cdot}4568$, as formerly.

In a similar way may be obtained an expression for the value of an annuity to the joint lives of x and y after the death of z, provided z die in n years.

The expression in ordinary notation is

$$\overline{_{n}|}\mathrm{A}_{x.y}-\overline{_{n}|}\mathrm{A}_{x.y.z}+{}_{n}a_{x\,n}a_{y}v^{n}\mathrm{A}_{x+n\;y+n}(1-{}_{n}a_{z}),$$

which becomes, when reduced to the primary items,

$$=\frac{l_xl_y\mathrm{A}_{x.y}d_z+vl_{x+1}l_{y+1}\mathrm{A}_{x+1.y+1}d_{z+1}+\text{\&c. to } n \text{ terms.}}{l_x.l_y.l_z}$$

$$=\frac{d_z\mathrm{N}_{x.y}+d_{z+1}\mathrm{N}_{x+1.y+1}+\text{\&c. to } n \text{ terms}}{\mathrm{D}_x.l_yl_z}$$

$$=\frac{\mathrm{N}_{\frac{1}{xy.z}}-\mathrm{N}_{\frac{1}{x+ny+n.z+n}}}{\mathrm{D}_{x.y}.l_z}$$

and $\therefore\ \mathrm{A}_{x.y}-\mathrm{A}_{x.y.z}=\dfrac{\mathrm{N}_{\frac{1}{xy.z}}}{\mathrm{D}_x.l_y.l_z}$.

Examples.

1. Value of annuity during the joint existence of 50 and 55 after the death of 60:—

$$\frac{\mathrm{N}_{\frac{1}{50.55.60}}}{\mathrm{D}_{50.55}.l_{60}}=\frac{19{,}073{,}497{,}490}{2071272\;.l_{60}}\qquad\begin{array}{lr} \lambda & 10{\cdot}2804304 \\ \kappa & \bar{7}{\cdot}6837634 \\ \kappa & \bar{4}{\cdot}4385408 \\ \hline \lambda & {\cdot}4027344 \\ \hline & 2{\cdot}5277 \end{array}$$

which, by the usual system, is

$$\begin{array}{lll} \mathrm{A}_{50.55} & = 9{\cdot}181 & \\ \mathrm{A}_{50.55.60} & = 6{\cdot}653 & \text{(per MS. tables)} \\ & \overline{\phantom{6{\cdot}653}} & \quad 2{\cdot}528 \end{array}$$

2. Value of an annuity during the joint existence of 50 and 55 after death of 60, provided that event take place in 7 years:—

$$\begin{array}{rrlr} \mathrm{N}_{\frac{1}{50.55.60}} & = 19{,}073{,}497{,}490 & & \\ \mathrm{N}_{\frac{1}{57.62.57}} & = 6{,}962{,}427{,}490 & & \\ \hline & 12{,}111{,}070{,}000 & = \lambda & 10{\cdot}0831825 \\ & \mathrm{D}_{50.55} & \kappa & \bar{7}{\cdot}6837634 \\ & l_{60} & \kappa & \bar{4}{\cdot}4385408 \\ \hline & & \lambda & {\cdot}2054867 \\ \hline & & \text{Answer} = & £1{\cdot}60504 \end{array}$$

while by the ordinary method the answer is

$$
\begin{array}{lrl}
\multicolumn{3}{c}{{}_{\overline{7}|}\mathrm{A}_{50.55}-{}_{\overline{7}|}\mathrm{A}_{50.55.60}+{}_{7}a_{50.55}v^{7}\,\mathrm{A}_{57.62}(1-{}_{7}a_{60})} \\
\mathrm{N}_{51.56} = & 19{,}016{,}722{\cdot}9 & \\
\mathrm{N}_{58{\cdot}63} = & 8{,}225{,}190{\cdot}3 & \\
\hline
 & 10{,}791{,}532{\cdot}6 = & \lambda\, 7{\cdot}0330819 \\
 & \mathrm{D}_{50.55} & \kappa\, \bar{7}{\cdot}6837634 \\
\hline
 & & {\cdot}7168453 = \quad 5{\cdot}21009 \\
\mathrm{N}_{51.56\,61} = & 61{,}081{,}368{,}129 & \\
\mathrm{N}_{58.63.68} = & 19{,}259{,}108{,}129 & \\
\hline
 & 41{,}822{,}260{,}000 = & \lambda\, 10{\cdot}6214075 \\
 & \mathrm{D}_{50} & \kappa\ \bar{3}{\cdot}2085105 \\
 & l_{55} & \kappa\ \bar{4}{\cdot}3900856 \\
 & l_{61} & \kappa\ \bar{4}{\cdot}4384592 \\
\hline
 & \lambda & {\cdot}6584628 = 4{\cdot}55473 \\
\hline
 & {}_{\overline{7}|}\mathrm{A}_{50.55}-{}_{\overline{7}|}\mathrm{A}_{50.55.60} = & {\cdot}65536 \\
 & \mathrm{N}_{58.63} & \lambda\, 6{\cdot}9151460 \\
 & \mathrm{D}_{57.62} = & \kappa\, \bar{7}{\cdot}6837634 \\
 & 1-{}_{7}a_{60} & \lambda\, \bar{1}{\cdot}3790570 \\
\hline
 & & \lambda\, \bar{1}{\cdot}9779664 = {\cdot}95053 \\
\hline
 & \text{Answer, as before} & . \quad \pounds 1{\cdot}60589
\end{array}
$$

And in the same way an expansion of the series for the value of an annuity to *x*, after the failure of the joint lives *y* and *z*, may be obtained.

The expression, in the ordinary notation, for the foregoing annuity payable if the joint existence fail in a term of *n* years, is

$$
{}_{\overline{n}|}\mathrm{A}_{x}-{}_{\overline{n}|}\mathrm{A}_{x.y.z}+{}_{n}a_{x}v^{n}\mathrm{A}_{x+n}(1-{}_{n}a_{y.z}),
$$

which, when reduced to the original elements, becomes

$$
=\frac{l_x\mathrm{A}_x d_{y.z}+vl_{x+1}\mathrm{A}_{x+1}d_{y+1.z+1}+\text{\&c. to } n \text{ terms}}{l_x.l_y.l_z}
$$

$$
=\frac{d_{y.z}\mathrm{N}_x+d_{y+1\,z+1}\mathrm{N}_{x+1}+\text{\&c. to } n \text{ terms}}{\mathrm{D}_x.l_y.l_z}
$$

$$
=\frac{\mathrm{N}_{\frac{1}{x.yz}}-\mathrm{N}_{\frac{1}{x+n.y+n\,z+n}}}{\mathrm{D}_x.l_y.l_z}.
$$

These examples sufficiently illustrate the applicability of the process of columnar calculation to the system of survivorship annuities, into which also other elements, such as the probability of marriage, &c., may be incorporated. The two examples involving three lives are given for the purpose of showing the direction of the general problem, tables for the various combinations of *three* lives being nearly impracticable.

Before concluding, I wish to make one remark of a practical nature, which these calculations may assist in explaining. I have observed, in many cases of contingent or survivorship annuities, that the annual premiums charged for the ages at the opening of the policies are *greater* than would be charged to the same persons for a similar transaction at their advanced ages. The Office has entered upon a risk for which the premium, as usually calculated, was only sufficient when continued during their entire existence or until the risk emerged; and if, by the privilege generally conceded to policy-holders of dropping the policy at any time, the Office only receive that average premium during the years when the risk is greater than the premium, it is obvious that the forfeiture of such a policy entails a loss to the Company. To give an illustration :—An annuity of £100 is granted to 60 after 40. The value of each year's risk, or, in other words, the

Value of an annuity to	60	provided	40	die in one year=	12·567
And the same to	61	„	41	„	12·946
	62	„	42	„	13·132
	63	„	43	„	12·936
	64	„	44	„	12 716
	65	„	45	„	12·301
	66	„	46	„	11·867
	67	„	47	„	11·244
	68	„	48	„	10·084
	69	„	49	„	9·645
	70	„	50	„	9·003
					&c. &c.

and the equal average premium which will cover these fluctuating risks is easily seen to be about 11·62. Thus—

$$100\frac{\text{A}_{60}-\text{A}_{66.40}}{1+\text{A}_{60.40}}=11{\cdot}62.$$

But it will be noted that, during the first seven years, the risk is greater than the premium receivable, and that the assured have at the end of that time the opportunity of opening a new policy of similar nature for a smaller premium than was demanded of them for the first policy. The premium which would in these cases recompense the Office for the risk (giving the assured the option of taking out a new policy at any subsequent date if he should be aware that it would be of any advantage to do so) may be determined by finding each year's risk during the earlier years of the assurance by means of the formula

$$_{\overline{1}|}\text{A}_x-{}_{\overline{1}|}\text{A}_{x.z}+{}_1a_x v^1\text{A}_{x+1}(1-{}_1a_z)=\frac{d_z\text{A}_x}{l_z},$$

and charging this sum; or by finding the yearly premium during 3 years or any other short period, for the value of an annuity to *x*, provided *z* died in that period—

$$\frac{N_{\frac{1}{x.z}}-N_{\frac{1}{x+n.z+n}}}{N_{x-1\,z-1}-N_{x+n-1.z+n-1}}.$$

Such a method would ensure that the Office was adequately remunerated for the risk it had undertaken.

Table for facilitating the Calculation of Survivorship Annuities during One Life, provided another die in n years (Carlisle 4 per Cent.).

To x after z.			To the joint of x and y after z.			
Ages.		$N_{\frac{1}{x.z}}$.	Ages			$N_{\frac{1}{xy.z}}$.
x.	z.		x.	y.	z.	
50	60	10,678,027	50	55	60	19,073,497,490
51	61	9,706,635	51	56	61	16,753,457,490
52	62	8,777,347	52	57	62	14,600,497,490
53	63	7,911,333	53	58	63	12,658,297,490
54	64	7,124,803	54	59	64	10,954,617,490
55	65	6,400,563	55	60	65	9,442,877,490
56	66	5,740,530	56	61	66	8,118,137,490
57	67	5,140,533	57	62	67	6,962,427,490
58	68	4,592,146	58	63	68	5,950,727,490
59	69	4,092,347	59	64	69	5,069,539,490
60	70	3,634,444	60	65	70	4,299,913,490
61	71	3,219,483	61	66	71	3,636,808,490
62	72	2,814,183	62	67	72	3,022,808,490
63	73	2,415,234	63	68	73	2,453,042,490
64	74	2,033,023	64	69	74	1,938,019,490
65	75	1,668,707	65	70	75	1,477,968,490
66	76	1,355,287	66	71	76	1,109,002,490
67	77	1,083,620	67	72	77	812,736,490
68	78	858,593	68	73	78	586,938,490
69	79	679,422	69	74	79	422,643,490
70	80	527,267	70	75	80	295,990,490
71	81	407,263	71	76	81	205,835,590
72	82	307,144	72	77	82	138,398,690
73	83	228,943	73	78	83	91,479,790
74	84	167,592	74	79	84	58,925,590
75	85	121,257	75	80	85	37,377,390
76	86	85,128	76	81	86	22,769,990
77	87	57,701	77	82	87	13,229,290
78	88	37,222	78	83	88	7,158,570
79	89	23,806	79	84	89	3,807,410
80	90	15,449	80	85	90	2,071,830
81	91	9,050	81	86	91	984,770
82	92	4,909	82	87	92	419,339
83	93	2,621	83	88	93	172,106
84	94	1,432	84	89	94	71,435
85	95	779	85	90	95	28,713
86	96	433	86	91	96	11,772
87	97	249	87	92	97	5,101
88	98	141	88	93	98	2,170
89	99	82	89	94	99	951
90	100	54	90	95	100	501
91	101	34	91	96	101	257
92	102	20	92	97	102	124
93	103	10	93	98	103	51
94	104	2	94	99	104	11